Surface of sand after failure of anchor

Behaviour of shallow inclined anchorages in cohesionless sand

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Summary
The aim of the present investigation is to examine the behaviour of small-scale shallow inclined anchorages buried in sand having a negligible cohesive strength. A theoretical approach was formulated connecting the variables involved and predicting the ultimate pull-out force of the anchorages.

Introduction
Earth anchorages are used to provide uplift resistance for a variety of structures, particularly the hanging roof in which the main structural member is the cable tie. Many of these structures rely on achieving cable anchorage using large dead weight foundations, but this may result in a great deal of the economic advantages of such a roof being lost.

The introduction of an earth anchor in place of the large dead weight foundations enables the vast bulk of the anchorage to be provided by the ground itself. Furthermore, this is particularly advantageous from the structural and economic viewpoint if serious ground disturbance can be avoided. This can be achieved by providing a small diameter reinforced concrete pile extending downwards from the ground surface and opening out at the bottom into an enlarged bulb to a specified size. This can be constructed in place, without disturbing the soil, by use of an expandable reaming device to provide the enlarged cavity at the bottom of the shaft before placing the reinforcing bars and filling with concrete.

So far, few earth anchors are commercially available, notable exceptions being the Webb Lipow and Harvey Anchors in the USA. Furthermore, although a not inconsiderable effort has been made to date by researchers in order to increase existing knowledge, the earlier papers (see reference 1 to 5 and 8) have been responsible for laying down many of the basic concepts, and much of this work has been concerned with the single vertical anchor subjected to axial loading. There appears to be no theoretical analysis to predict the behaviour of inclined anchors, and little in the way of reported tests on anchors of this type, either full size or small scale.

This view is endorsed by Aleksander S. Vesic in the following statement, "..... It is significant to note that the only known experimental investigation of the effect of load inclination, made with plates in fine sand showed....." (12 refers to Kanayan's paper) Also, more recently, by W. J. Larnach who wrote, "..... Reported work on anchors, inclined rather than at the vertical is more limited....." This lack of existing work is most unfortunate as the construction industry has a need for tractable theoretical material, and such work can be difficult to prepare when there are few published full scale tests for use in analysis.

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Previous work

Investigation of the mode of failure of a system of this type has shown that there are two phases during failure, other than the structural failure of the anchor itself, both of which may occur depending on the value of the ratio of the depth of embedment of the anchor to the size of the anchor bulb. For larger values of this ratio, i.e. "deep" anchors, failure occurs due to "tunnelling" of the anchor through the soil with little or no displacement of the surrounding soil mass, and with no accompanying surface effects. This process continues until the depth of embedment has decreased sufficiently, i.e. "shallow" anchors, to displace a cone of soil at the free surface. Failure is accompanied by radial and circular cracking, in the case of a vertical anchor, on the soil surface. Similar characteristics occur for inclined anchors although the surface cracks are elliptical, as opposed to circular in the case of a vertical anchorage.

The distinction between "shallow" and "deep" anchor action has been made by various authors, and the ratio of depth of embedment to anchor diameter at the "critical" depth has been reported to vary between 4 and 6, depending on the soil conditions and form of anchor bulb or plate.

Methods of predicting the ultimate pull-out resistance of the anchorages have dealt only with anchorage systems with vertical axes and include earth weight theories which consider the pull-out capacity to be the weight of soil inside an assumed failure surface; refinements take into account the shearing resistance of the soil along an assumed failure surface and, in the case of cohesive soil, the cohesive strength also. Several authors have based their assumed shape of failure surface on the behaviour of an anchor loaded in the usual way, but tested adjacent to the transparent face of a tank, thus enabling the failure mechanism to be studied visually.

Of the foregoing methods, adequate correlation is given between Balla's theory and his small scale model tests. However, in all cases, the methods are not applicable to "deep" anchors, and yield results for the ultimate pull-out force which are well on the unconservative side. Attempts to analyse anchors of this type have been made by Mariopolski and Vesic by evaluating the work done in expanding a spherical, or vertical cylindrical, cavity in an infinite soil mass.

Baker and Kondner carried out a dimensional analysis and obtained the empirical constants from the results of their model tests. The resulting equations distinguished between the behaviour of "deep" and "shallow" anchors and some correlation was shown in the case of tests on full-scale anchors.

Theory

The following theory applies to the ultimate pull-out capacity of a shallow inclined anchorage embedded in a soil with both frictional and cohesive properties.

Centre—Underside of tank showing arrangement of neoprene blocks and vibrator

Right—Diagrammatic sketch showing arrangement of apparatus
The anchor consists of a thin circular anchor plate to which the pull-out force is symmetrically applied by means of a central point load at right angles to the plane of the anchor plate.

The diagram, fig. 1, shows a vertical section through the failure zone corresponding to the major axis of the ground surface failure ellipse. The shape of the failure surface is considered to be an arc of a circle perpendicular to the anchor plate and making an angle of \((\pi/4 - \phi/2)\) with the horizontal free surface. This angle corresponds with the slope of the failure lines in the passive Rankine state in the soil, and examination of the zone of failure at the surface which arises from this assumption corresponds fairly well with that observed by Kanayan for normal inclinations.

Considering a typical sector of the failure zone located at an angle \(\xi\) from the vertical section shown above the relationships between \(\psi, \omega, \psi\) and \(\xi\) is given by:

\[
\tan \psi = \tan \xi \sin \omega \tag{1}
\]

\[
\sin \psi = \sin \xi \cos \omega \tag{2}
\]

The angle formed between the curvilinear surface of sliding and the horizontal surface of the soil is \((\pi/4 - \phi/2)\). Thus in the plane of a typical sector the angle between the curvilinear surface of sliding and the horizontal soil surface is given by, using equations (1) and (2):

\[
\tan \theta = \tan \xi + \tan^2 \psi \sin^2 \xi \tag{3}
\]

Simplifying the failure surface by replacement of the curvilinear surface by a singly curved surface defined by the angle \(\delta \xi\), the volume of a typical sector of the failure zone subtending an angle \(\delta \xi\) is given by:

\[
\delta V = \frac{d^2 D}{6} (L + - \tan \theta) \cos \eta \tag{4}
\]

\[
\delta A = \frac{\cos \eta \tan \theta - \sin \eta}{2} \delta \xi \tag{5}
\]

Also area of sector of anchor plate:

\[
\delta A = \frac{d \cos \eta - \frac{D}{2}}{2} \delta \xi \tag{6}
\]

Now as the earth pressure force at a depth \(z\) in the soil is given by:

\[
p_e = K_e \gamma z
\]

where \(K_e\) = coefficient of earth pressure at rest, and \(\gamma\) = density of soil.

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<th>Expt.</th>
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**Fig. 2.** Graph of \(P_U/\gamma D^3/H/D\)

**TABLE 1**—Comparison of theory with experimental results

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TABLE 2—Comparison of theory with Baker and Kondners' experimental results

<table>
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</tr>
<tr>
<td>4.3</td>
<td>49.0%</td>
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Total radial force including effect of weight component:

\[
S = \delta \xi \left\{ \frac{K_o \gamma d L^2 \cos^2 \psi}{2 (L + \frac{\tan \theta}{\tan \phi})} \right. \\
+ \gamma \delta V \sin \omega \cos \omega \\
\left. \frac{1}{8} \gamma D^2 \sin^2 \omega \cos \omega \cos^2 \psi (L + \frac{\tan \theta}{\tan \phi})^2 \right. \\
- \frac{1}{8} \gamma D^2 \sin^2 \omega \cos \omega \cos^2 \psi \tan^2 \theta \\
- \frac{D}{2} \left( L + \frac{\tan \theta}{\tan \phi} + \frac{L}{3} \right) \right) 
\]

Now if we consider the anchor plate reaction on the adjacent soil as \( P \) acting at an angle \( \phi \), we obtain, by resolving all forces in the plane of the sector in a direction perpendicular to the resultant soil reaction on the curved surface of the sector:

\[
P \sin \theta = \gamma \delta V \cos \omega \sin (\phi + \theta - \eta) + \\
C_\lambda \delta A \cos \phi + C_n \delta A^2 \cos (\theta + \phi) - \\
S \cos (\theta + \phi - \eta) 
\]

All the required unknown quantities are given by equations (1) to (7); \( C_\lambda \) and \( C_n \) are the coefficients of cohesion for the respective surfaces.

Thus the ultimate pull-out force is given by:

\[
P_u = \sum P \cos \phi \\
\xi = \frac{2w}{2 \pi} \\
\xi = o 
\]

where \( P \) is found from equation (8).

This theory was programmed for solution on a 1900 Series computer. The size of each sector was defined by taking \( \delta \xi \) as \( 1^\circ \).

Experimental procedure

The model anchor consisted of a tie rod connected to a thin circular anchor plate; the diameter of the anchor plate was varied from 18 mm to 51 mm (7/8" to 2""). The anchor was embedded in dry sand...
contained in a box of internal dimensions 812 mm x 635 mm x 432 mm (32 in x 25 in x 17 in) deep. This was considered large enough to prevent interference with the failure zone of anchors embedded to a depth of approximately 254 mm (10 in) and with angles of inclination of 0 deg, 15 deg, 30 deg and 45 deg with the vertical. The physical properties of the sand were density (\(\gamma\)) = 1730 kg/m\(^3\) (108 lb/ft\(^3\)) and angle of internal friction (\(\phi\)) = 40 deg.

After embedding the anchor compaction of the sand was achieved by vibration, and then a gradually increasing force was applied to, and in the same line as, the tie rod until failure occurred. This was defined as occurring when the applied force was sufficient to cause a disproportionate movement of the anchor; this force was either the same as the pull-out force or only fractionally less than it.

As was anticipated, by studying the surface of the sand after failure of the anchor, the failure pattern on the surface was elliptical in shape when the anchor was inclined and circular when vertical.

**Experimental results and discussion**

A total of forty-nine experiments were carried out. These mainly comprised tests on inclined anchors but a number of tests were also made on vertical anchors. A sample of twenty-four tests are shown in table 1. Also shown are the theoretical results based on the theoretical work outlined in the previous section.

(i) Examination of figure 2, which shows Pu/\(y\) D\(^2\)/H/D, demonstrates that for values of H/D \(<6\) and H/D \(>6\) the form of relationship between the dimensionless groups is different. This is consistent with the results obtained by Baker and Kondner for vertical anchors. Thus the concept of "critical depth" and the distinction between "deep" and "shallow" failure applies to both vertical and inclined anchors. The theoretical results apply only for "shallow" anchors, i.e. for H/D \(<6\) and it is instructive to compare the theoretical results with the practical results for both the present experimental work (table 1 and figure 2) and also with the results of Baker and Kondner, table 2.

Good agreement is shown in the case of results where H/D \(<6\) but for values of H/D greater than this (marked with an asterisk), the theoretical results generally overestimate the experimental results. This is to be expected as an allowance has been made for "tunnelling" which reduces the size of the failure cone.

(ii) Fig. 3 again shows Pu/\(y\) D\(^2\)/H/D. In this case, however, comparison is made of the results on the basis of inclination of the anchors. For example the pull-out force when vertical is not greatly different to that of a similar inclined anchor embedded at the same vertical depth. The maximum percentage difference decreases as the H/D ratio increases.

It is interesting to compare this result with that obtained by other authors. Kanayan\(^1\) found that for shallow inclined anchors the pull-out force is considerably increased over that for vertical anchors embedded at the same depth. These results together with the corresponding theoretical results, are shown in table 3. For example in the case of an anchor inclined at 30 deg to the vertical there is an increase in capacity of 22 per cent which is considerably larger than that found in the present tests. This increases to a 49 per cent greater capacity in the case of an anchor inclined at 45 deg. However, Trofimenkov and Mariupol'ski\(^2\) found that there is generally very little difference in capacity between vertical anchors and those inclined at 45 deg.

This apparent difference may be explained by the presence of cohesion which would play the more important part in pull-out resistance as the surface area of the failure cone increases. This would be the case in failure of anchors with an inclined axis.

Close agreement can be seen between theory and experiment by examination of table 3. As all Kanayan's experiments\(^1\) were carried out using shallow anchors the theoretical work is applicable in all cases. It is anticipated that, as the anchor shaft diameter used in Kanayan's work was only 38 mm (1.5 in), this would not invalidate the use of the present theory which, of course, makes no allowance for a shaft.

(iii) In fig. 4 the relationship between the ultimate pull-out load and the diameter of the anchor plate is shown for various depths of embedment. Also shown is the H/D = 6 line. It can be seen that as the H/D ratio decreases the capacity of the anchor tends to be less affected for any given depth. For example when considering H/D \(<6\) (approximately) the capacity of the anchor is not greatly affected by fluctuations in the diameter of the anchor plate. However, for values of H/D \(>6\) the capacity of the anchor increases considerably when the diameter of the anchor plate is increased at a given depth. Thus it would be much more efficient from the point of view of pull-out capacity to, say, double the diameter of the anchor plate for a "deep" anchor than a "shallow" anchor.

(iv) Table 4 compares the results of Baker and Kondner's field tests\(^1\) with the theoretical predictions from the work in this paper. For anchor No. 1, which is a "shallow" anchor, the theory underestimates the actual test result but the dif-
erence is thought to be due to the presence of cohesion due to the grout used to stabilise the sand during the reaming operation.

In the case of anchor No. 2 which is a "deep" anchor the theory, as expected overestimates the pull-out resistance. The error is apparently reduced, however, due again to the effect of the grout.

Theoretical work carried out during the present investigation into the effect of cohesion has shown that even the presence of a small cohesive force can considerably increase the pull-out capacity of an anchor. For example, if the cohesive force was neglected in the calculation of the theoretical results in Table 3a the pull-out capacities would be reduced by something of the order of 25 per cent.

Practical benefits
In the design of anchorages systems, the present practice is to carry out an assessment of the pull-out capacity using established empirical formulae. This may be supplemented by full scale tests at the site of the proposed embankments which, though expensive, provide valuable information. However, the ability to make a theoretical assessment is essential if present prejudice against the use of these anchors is to be overcome. (This was clearly demonstrated by the refusal of the Olympic Building Authority to allow ground anchors to be used to anchor the Munich Olympic roof, instead, insisting on large concrete deadweight anchors thereby considerably escalating the cost of the project). Furthermore, an added advantage of the theoretical approach is that it enables the interaction zones between anchor groups to be predicted, and consequently facilitates an optimisation of the design of such arrangements.

In conclusion, it is evident that the rapid development of anchor techniques in construction over the last decade, in a wide range of applications, has shown that practice has outstripped theory. It is thought that the theory developed in this publication has made a contribution towards closing this gap.

Conclusions
Due to apparent paucity of work in the field of inclined anchors the work in this paper may be regarded only as an initial investigation into the theoretical and practical aspects of their behaviour. This may be regarded as a limited part of a continuing investigation.

Several points have emerged which may be summed up as follows:
(i) The concept of a critical H/D value differentiating between "deep" and "shallow" anchors may be applied to both vertical and inclined anchorages.
(ii) Ultimate capacities of inclined and vertical anchorages at the same depth of embedment are approximately the same for non-cohesive soils.
(iii) The proposed theory is valid for "shallow" anchorages, both vertical and inclined, and good correlation is shown between predicted results and experimental work.

Future work
The following areas of work require further investigation:
(i) Further tests on inclined anchorages in a variety of different soil types.
(ii) Assessment of the effect of anchorage shafts on pull-out resistance.
(iii) Anchorage groups.
(iv) Effect of different types of loading.
(v) Recommendation of a design procedure with adequate safety factors covering both working loads and anchor movement.

### TABLE 3—Comparison of theory with Kanayan's experimental results for shallow vertical and inclined anchors

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\[ \psi = 32 \text{ deg}; \quad \gamma = 0.059 \text{ lb/in}^2; \quad c = 0.3 \text{ lb/in}^2 \]

### TABLE 4—Comparison of theory with Baker and Kondner's field tests

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<tr>
<td>[ \psi = 37 \text{ deg}; \quad \gamma = 0.0648 \text{ lb/in}^2 ]</td>
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**Notation**
The notation used in the theoretical work is as follows:

- \( \psi \) = inclination of anchor axis to vertical, i.e pull-out angle.
- \( \xi \) = horizontal angle locating a typical sector of the failure zone—measured from major axis of surface failure ellipse.
- \( \delta \) = angle subtended by typical sector of failure zone.
- \( \omega \) = angle of inclination of typical sector of failure zone relative to vertical axis.
- \( \eta \) = angle of inclination of pull-out axis to plane perpendicular to failure zone sector.
- \( \phi \) = angle of internal friction of soil.
- \( C \) = coefficient of cohesion of soil.
- \( R \) = radius of failure surface.
- \( D \) = diameter of anchor plate.
- \( \alpha \) = angle between curvilinear surface of sliding and horizontal surface in plane of failure sector.
- \( \theta \) = angle defining location of simplified failure surface.
- \( \alpha \) = area of sector of failure zone.
- \( \Delta A \) = area of curved surface of sector of failure zone.
- \( d \) = soil surface distance of failure zone to pull-out axis.
- \( \Delta A \) = area of sector of anchor plate.
- \( \Delta W \) = weight of sector of failure zone.
- \( \gamma \) = soil density.
- \( K_0 \) = coefficient of earth pressure at rest assumed constant with depth.
- \( H \) = vertical depth of centre of anchor plate below soil surface (\( = L \cos \psi \)).
- \( S \) = radial resultant of earth pressure forces and weight component.
- \( P \) = anchor plate reaction on soil.
- \( P_D \) = ultimate pull-out force.
- A, B = subscripts.

**References**