A simple approach to slope stability

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Introduction

SLOPE STABILITY analysis has for some years been regarded as an area where computing techniques can readily assist the engineer. The computer enables hundreds of potential slip surfaces to be analysed in a very short space of time and can subsequently search for the minimum factor of safety — all at very reasonable cost.

Unfortunately the ease with which data can be fed in to commercially available programs with the satisfaction of almost immediate solutions being churned out has led in certain cases to the engineer no longer appreciating the significance of the analysis being undertaken.

In all applications of slope stability analysis a 'hand' calculation should be carried out preferably before the computer is used. In this way the effects of slope geometry, assumed water levels and properties assigned to the strata can be studied not only in relation to the overall factor of safety but also as they affect the forces on individual 'slices' within the analysis. For example the soil parameters and groundwater table assumed at the toe of the slope may well be shown to be the major factor in controlling the stability of that slope. The more complex the soil strata the more important it is that a hand calculation is carried out since the computer program often makes over-riding assumptions not fully appreciated by the operator.

Simple solution

An effective stress solution for slope stability which by its simplicity lends itself to hand calculation for both circular and non circular slips is developed in the Appendix. The solution is based on conventional shear strength theory. Where horizontal stresses in the ground can be estimated or measured the equation may be applied in its full form (equation 7). The horizontal earth pressure coefficient (K) is dependent on the previous stress and geological history of the strata and the method of construction of the slope; it may also be time-dependent. In a normally consolidated soil slope K might vary typically from a value corresponding to the 'active' state near the crest of the slope (reducing to zero if a tension crack is present) to something near the 'at rest' state below the toe. However, the equation is not particularly sensitive to the values of K selected (the factor of safety assuming $K=0$ is approximately 5 – 15% less than that calculated for $K=0.5$) and for routine calculations it may be conservatively assumed that $K=0$. This has the effect of ignoring the contribution to the shearing resistance made by horizontal forces on the slip surface and gives the simple equation:

$$F = \frac{\Sigma[c'\,b \sec\alpha + (W - ub)\cos\alpha \tan\phi']}{\Sigma W \sin\alpha}$$

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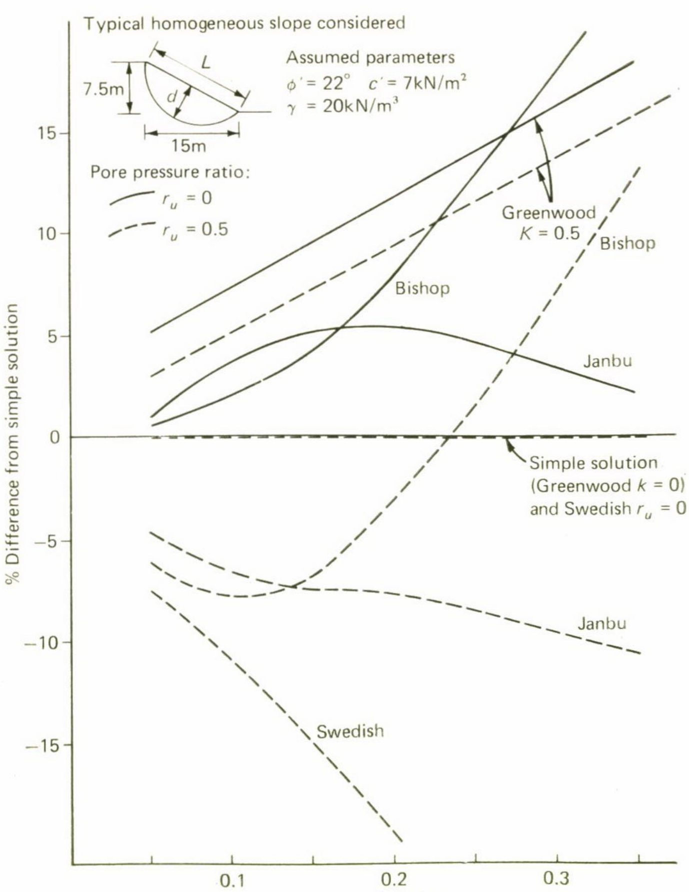
Notation

b	width of soil slice (m)
c'	effective cohesion of soil at slip surface (kN/m^2)
F	factor of safety
h	average height of soil slice
K	ratio of horizontal to vertical effective stresses
l	length of slip surface of soil slice; assumed $b = l\cos\alpha$ (m)
P'	effective normal force (kN)
r_u	pore pressure ratio at slip surface $= \frac{\text{pore water pressure}}{\text{total overburden pressure}}$
τ_s	shearing resistance of soil slice
u	pore water pressure at slip surface (kN/m^2)
W	total weight of soil slice (kN)
α	base inclination of soil slice (negative at toe)
γ	total density
γ'	submerged density
σ_n'	effective normal stress on base of soil slice
σ_v'	effective vertical stress on base of soil slice
ϕ'	effective friction angle of soil at slip surface



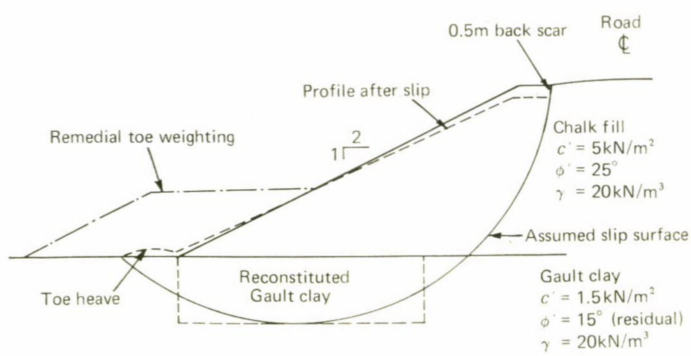


Fig. 1 (left). Comparison between "simple" solution and other solutions for a typical slope with circles of varying depth

Fig. 2 (above). Application of simple analysis. Foundation failure beneath a marly chalk embankment near Cambridge or in terms of the pore pressure ratio, r_u

$$F = \frac{\Sigma\,[c'\,b \sec\alpha + W\,(1-r_u)\cos\alpha\tan\phi']}{\Sigma\,W\sin\alpha}$$

where $r_u = \dfrac{ub}{W}$

Comparison with other solutions

This equation is identical to the Swedish equation (Fellenius, 1936) modified as suggested by Turnbull & Hvorslev (1967) to resolve only the effective vertical force on each slice of the analysis.

The simple solution is compared in Fig. 1 with the full equation (assuming K = 0.5) and with the Bishop Simplified (Bishop, 1955), Swedish and Janbu (Janbu *et al*, 1956) solutions for a typical slope with circles of varying depth and for both $r_u = 0$ and $r_u = 0.5$.

The effect of assuming $K = 0$ is conservative reducing the calculated factor of safety by about 10%. The difference between the author's simple solution and the Bishop Simplified is generally less than 15% with the simple solution giving higher values for shallow slips and Bishop giving higher values for deeper slip circles. The author's 'full' and 'simple' equations give consistent results for both shallow and deep slip surfaces whereas the Bishop and Swedish equations tend to become 'unstable' for deep slip surfaces.

Practical application

In practice, when the geotechnical engineer is presented with a stability problem, the soil strata, strength parameters and pore water pressures are rarely known with any accuracy.

The application of a simple equation such as that described enables the significance of the variables and their effect on the factor of safety to be studied. There is little point in applying more sophisticated methods of analysis when the differences in calculated factor of safety due to the various assumptions far exceed the differences due to the method used and indeed the more complex solutions may obscure the effects of changed parameters.

Many stability problems involve reinstatement of slipped soil masses. The logical approach is to analyse the failure and compare the existing factor of safety with that after the proposed remedial measures have been taken. It is the increase in factor of safety that is usually more relevant than the absolute values.

Example of application

In this example the above philosophy was applied to a slip which occurred approximately 18 months after construction of a 8m high marly chalk embankment on a Gault Clay foundation (Fig. 2). The Gault Clay had been excavated and recompacted to a depth of 3m beneath the toe prior to construction of the bank in an attempt to destroy existing shear surfaces known to be present in the Gault in this region.

The top of the embankment settled by about 0.5m at the edge of the road and heave occurred at and beyond the toe indicating a foundation failure. The geometry of the slip suggested that it passed along the base of the dig-out zone.

A rapid assessment of remedial measures was required because in addition to the road itself main trunk services were at risk.

Typical parameters were assumed as indicated on Fig. 2 based on previous design testing and analysis. Residual strength parameters were assumed for the Gault as movement had already occurred. Initially a high water table was assumed within the bank because water was observed in the service manholes at the top of the bank. The resulting factor of safety was unrealistically low (0.76). The water table was then assumed 0.5m below the original ground surface giving a more sensible factor of safety of 0.86. (Subsequent piezometer installations confirmed the lower water table).

Remedial measures were then considered. With little scope for lowering of the water table in the foundation, toe weighting appeared the most feasible solution and analysis of the slip with a 3m high underdrained berm showed a factor of safety of 1.28. A reassuring 50% increase on this particular assumed slip surface. Remedial works were undertaken on this basis and the movement on the slope successfully arrested.

The revised slope geometry was checked for other possible critical circles using the commercial programs, SLIPSYST and CIRCA. The minimum factor of safety was found to be a satisfactory 1.5 by the Bishop method and 1.1 by the Swedish method.

Conclusions

When analysing the stability of a soil slope a hand calculation is beneficial in understanding the potential forces acting on the slope and for appreciating the sensitivity of the calculated factor of safety to the parameters selected. The commercial computer program is a valuable aid in determining the most critical slip surfaces once there is a basic understanding of the slope and parameters.

A simple equation for slope stability analysis has been developed. Its application has been demonstrated for routine calculations where often the strata, strength parameters and water pressures cannot be precisely defined and where the effects of variations in the parameters are to be studied.

The simple equation is readily applied by hand and results are shown to be comparable with other accepted, but more complex, methods of analysis.

Acknowledgements

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APPENDIX

Development of simple equation for slope stability

The factor of safety of a slip surface is most simply defined as the shear resistance available along the slip surface divided by the shear stress along that slip surface (Lambe & Whitman, 1969). By summation for vertical soil slices, as shown in Fig. 3, the following expression is obtained for the factor of safety of a slip surface in a soil having both c' and ϕ' values

$$F = \frac{\Sigma\ (c'\ l + P'\ \tan\ \phi')}{\Sigma\ W\sin\alpha} \quad \ldots\ (1)$$

This expression may be applied to both circular and non circular analysis.

Proposed effective stress stability equation

It is necessary to determine appropriate values of the effective normal stress on the slip surface, P', for each soil slice as shown in Fig. 4.

P' may be determined by consideration of a Mohr's circle diagram for the soil element at the base of the slice, as shown in Fig. 4. The horizontal effective soil pressure is given by $K\sigma_v'$ where K is

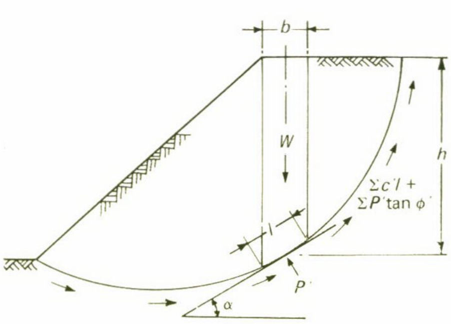
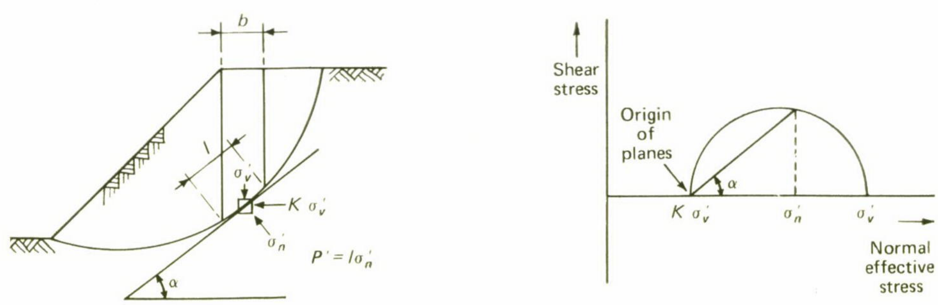


Fig. 3. Slip circle analysis by consideration of soil slices



(a) Slip surface

(b) Mohr circle of effective stress

Fig. 4. Consideration of soil element on potential slip surface at base of slice

the ratio of the horizontal to vertical effective stresses at the base of the slice. Possible rotation of the principal stresses due to the inbalance of forces within the slope is ignored. This may be compared to ignoring inter-slice forces in other derivations of equations for slope stability, and this has been shown to be generally insignificant on circular slip surfaces (Bishop, 1954; Whitman & Bailey, 1967; Bishop & Morgenstern, 1960).

From the Mohr circle of effective stress (Fig. 4):

$$\sigma_n' - K\sigma_v' = (\sigma_v' - K\sigma_v')\cos^2\alpha \ldots \quad (2)$$

re-arranging

$$\sigma_n' = \sigma_v'\,(\cos^2\alpha + K\;[1 - \cos^2\alpha])$$

$$\sigma_n' = \sigma_v'\,(\cos^2\alpha + K\sin^2\alpha) \quad \ldots (3)$$

but

$$P' = l\sigma_n'$$

and

$$\sigma_v' = \frac{W - ub}{b} \quad \text{(neglecting inter-slice forces)}$$

therefore

$$P' = \frac{l\,(W - ub)}{b}(\cos^2\alpha + K\sin^2\alpha) \quad \ldots (4)$$

Providing the width of slice is not large it is usually assumed, without significant loss in accuracy, that

$$l = \frac{b}{\cos\alpha} \quad \ldots (5)$$

substituting for l

$$P' = \frac{W - ub}{\cos\alpha}(\cos^2\alpha + K\sin^2\alpha)$$

or

$$P' = (W - ub)\cos\alpha\,(1 + K\tan^2\alpha) \quad \ldots (6)$$

substituting for P' and l (eqs. *5* and *6*) in eqn. *1*, the factor of safety of the slip surface is obtained:

$$F = \frac{1}{\sum W\sin\alpha}\sum[c'\,b\sec\alpha + (W - ub)\,(1 + K\tan^2\alpha)\cos\alpha\tan\phi'] \quad \ldots (7)$$

or in terms of the pore pressure ratio

$$r_u = \frac{u}{\gamma h} = \frac{ub}{W}$$

$$F = \frac{1}{\sum W\sin\alpha}\sum[c'\,b\sec\alpha + W\,(1 - r_u)\,(1 + K\tan^2\alpha)\cos\alpha\tan\phi'] \quad \ldots (7a)$$

A new method for measuring the impact energy of a piling hammer *(continued from page 38)*

down the bar (similar to the ram's image moving down the slit), a continuous trace is plotted on the paper.

The result of using the attachment is therefore a polaroid print with a continuous trace of the ram-displacement against time. If suitable scales can be included for the two axes, the impact velocity may be deduced from the gradient of the trace at impact.

Practical application

A small mark (20mm × 40mm) is printed on the ram as a reference point for monitoring the ram movement. A sequence of marks, at regular intervals, is printed on the hammer leaders or guide frame. By suitable positioning of the camera, the ram mark is aligned to run adjacent to the frame marks, and their image is positioned in the fixed slit (Plate 1).

During the ram's descent the d.c. motor is manually triggered to move the polaroid print across the slit. The ram's movement is recorded for 0.4 seconds, and during this time flashes of light from the LED are exposed to the print. These give a precise time scale for the displacement/time graph, and can be clearly seen in Figs. 4 and 5.

The image of the frame marks is a series of stationary points in the slit. Their trace on the polaroid print is therefore a sequence of horizontal lines. Since the spacing of the marks on the hammer frame is known, these lines give the precise displacement scale for the ram movement.

The image of the ram mark is a single point in the slit which moves down as the ram descends. Its trace is therefore a smooth curve as it accelerates with time. This can be seen in Fig. 5.

Results

Two results are presented in Figs. 4 and 5. Fig. 5 shows a hammer impact recorded in the field. The camera was positioned 60m from the hammer, and the frame marks were spaced at 100mm. The calculated impact velocity was 4.6 metres/second.

Fig. 4 shows the result of a laboratory test used to assess the accuracy of the method. A signal generator was used to oscillate the spot of an oscilloscope in a vertical line, at a frequency of 2.86Hz, and an amplitude of 69mm (a constant velocity of 0.394m/sec). Using the streak camera, the spot's velocity was measured at 0.408m/sec, giving an error of 3.5%. This error was due to the inaccuracy of measuring directly from the polaroid print. Higher resolution may be achieved by re-photographing and enlarging the print.

Conclusions

A reliable, accurate and remote method of measuring the working efficiency of a piling hammer has been described. The only requirement is a clear view of the ram. The method is as accurate as alternative methods; it is however quicker and easier to use.

BOOKS & Trade Literature

Pile Hammer Brochures. Two new brochures have been published by MKT Geotechnical Systems, Box 793., 100 Richards Avenue, Dover, NJ 07801, USA. A 4-page brochure gives details of four models of convertible diesel pile hammers which are basically of double-acting design but with a few modifications can be changed to single-acting. These range from the DA-15C with an energy rating of 913-1 134kgm to the DA-55C with a rating of 4 314 to 5 282kgm.

Two new compressed-air or steam operated single-acting pile hammers are described in a 2-page brochure. These are the MS-350, with a ram weight of 3.5tonnes and stroke variable between 300mm and 1.2m, and the MS-500, with a ram weight of 5tonnes and stroke variable between 600mm and 1.2m. A claimed feature of these hammers is that the ram weight represents about 65-70% of the total weight, resulting in a lower overall weight relative to the energy output.

Tunnels — Planning, Design, Construction, Vol. 2. by T. M. Megaw & J. V. Bartlett. Published by John Wiley & Sons Ltd., and available from their Distribution Centre, Shripney, Bognor Regis, West Sussex, PO22 9SA. 235mm × 160mm. 321pp; illus.; price £25.00.

Volume 2 of this work contains chapters on cut-and-cover and submerged-tube tunnels, shafts and caissons, a little geology and ground treatment and sections on the three principle types of transportation tunnel. A final chapter goes into more detail of the ventilation needs of vehicular tunnels.

The book tends to emphasise British practise and may be regarded as a tunnel designer's check-list; a description of the many problems that beset a tunnel planner or designer rather than as a guide to their solution. The bibliographies which accompany each chapter and the full (40p) bibliography at the end of the book are excellent.

The book may be recommended to the post-graduate student requiring a general introduction to tunnelling and would be a valuable source of background material to a newcomer in a tunnel engineer's office.

The Book is the simple title of a well-illustrated 20-page booklet from Winn & Coales (Denso) Ltd., Denso House, Chapel Road, London SE27 OTR. This is intended as a guide to the range of tapes, mastics and compounds developed by Winn & Coales in the corrosion prevention and sealing fields, as well as the special services offered, which include technical advice, training courses and design services.