Finite element analysis of soil containing vertical drains

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Introduction

The construction of embankments over soft, compressible soils can cause considerable problems to the geotechnical engineer. Expedients such as reinforcement, vertical drains or stage construction are often necessary to achieve an efficient, economical design. The analysis of such embankments becomes complex and requires the use of a numerical technique such as the finite element method.

Although the use of vertical drains has become widespread in embankment construction the finite element analysis of such problems has only occasionally been reported. The reason for this may be the difficulty in equating the plane strain embankment analysis with the essentially axisymmetric consolidation behaviour of the soil surrounding a single vertical drain.

In this paper a method for the plane strain finite element consolidation analysis of soil improved by vertical drains is presented. The consolidation of axisymmetric and plane strain unit cells, representing the soil around a single drain, are compared for a normally consolidated clay deposit. The clay formed part of the subsoil at a well documented construction project which has been analysed in full plane strain. Results from the finite element prediction are compared with published observed behaviour.

All the finite element analyses were performed using a modified version of the program CRISP (Gunn and Britto 1984).

Porto Tolle case history

A thermal power plant constructed on the Po river delta at Porto Tolle required the placement of several large steel tanks. To limit settlement after the placement of these tanks pre-loading embankments were constructed and vertical drains installed in the subsoil. The amount of vertical drains used at the site (= 1 700 000m) justified a comprehensive site investigation followed by the construction of a trial embankment beneath which several types of vertical drain were installed. The trial was extensively monitored and used to assess each drain type.

The quality of the field and laboratory data (Hansbo et al 1981) allowed the input parameters for the finite element analysis to be chosen with unusual confidence. Also, the large amount of instrumentation and sustained monitoring of the trial embankment enabled meaningful comparisons between finite element and observed behaviour to be made. The Porto Tolle case history therefore provided an excellent opportunity of testing the proposed analysis procedure and assessing the quality of the finite element analysis.

Subsurface conditions

The soil profile at Porto Tolle, Figure 1, was subjected to exceptionally intensive investigations (Jamiolkowski et al 1980) which showed the main clay stratum to be a young, normally consolidated clay. The clay behaviour was simulated using the modified Cam-clay model (Roscoe and Burland 1968) for which the four basic parameters were derived from the available literature (Garassino et al 1979; Jamiolkowski and Lancellotta 1984). The parameters used in the analysis are summarised in Figure 1.

Trial embankment construction and geometry

The embankment, Figure 2a, was divided into four areas beneath which different drain types were installed, Figure 2b. All the present analysis concentrated on the Geodrain area. The design height was achieved in 3.5 months after which the subsoil was allowed to consolidate for a further 10 months.

Axisymmetric versus plane strain consolidation analysis

The vertical drains at Porto Tolle were installed in a triangular grid at a spacing of 3.80m, Figure 3. The area of influence of each drain can be approximated by a circle, radius 2m, such that for analysis of a single vertical drain axisymmetry is appropriate. However, when a full plane strain analysis is performed the drains cannot be analysed as axisymmetric but must be idealised as continuous trenches, Figure 4. It is therefore necessary to find a logical basis for the spacing of the drains in a plane strain analysis.

![Figure 1: Porto Tolle soil profile and finite element material parameters.](image1)

![Figure 2: The Porto Tolle trial embankment: (a) plan view; (b) cross section.](image2)

![Figure 3: Vertical drain layout.](image3)

![Figure 4: Plane strain analysis.](image4)
Matching procedure

Under an instantaneous step loading, the average degree of consolidation, \( \bar{U}_b \), on a horizontal plane at depth \( z \) and time \( t \) of an axisymmetric unit cell, neglecting the effect of well resistance and smear, has been shown by Hansbo (1981) to be
\[
\bar{U}_b = 1 - e^{-St_h/b} \tag{1}
\]
where \( T_h = C_h t / 4R^2 \) is the time factor for radial drainage (\( C_h \) is the horizontal coefficient of consolidation, \( t \) is the time from application of load and \( R \) is the radius of the unit cell). The factor \( \mu \) is defined as \( \mu = \ln(n) - 3/4 \) where \( n = D/d_w \) (\( D \) is the diameter of the unit cell and \( d_w \) is the equivalent drain diameter).

Hansbo’s theory can easily be extended to consider a plane strain unit cell, Appendix A, and Equation 1 can be applied with \( \mu = 2/3 \) and \( T_h = C_h t / 4R^2 \), where \( B \) is the plane strain unit cell width.

In order for the rate of consolidation to be equivalent in plane strain and axisymmetric unit cells the average degree of consolidation must be the same at every time. Hence for perfect matching
\[
\bar{U}_{pl} = \bar{U}_{ax} \tag{2}
\]
where the subscripts \( pl \) and \( ax \) represent plane strain and axisymmetric conditions respectively. From Equation 1
\[
T_{pl} = T_{ax} \tag{3}
\]
or
\[
\mu_{pl} = \mu_{ax} \tag{3}
\]
\[
C_{pl} = C_{ax} \tag{4}
\]

The soil properties are assumed to be the same in both axisymmetry and plane strain, then rearranging Equation 4 and substituting appropriate expressions, the equivalent plane strain drain spacing is
\[
B = R \sqrt{3/2(\ln(n) - 3/4)} \tag{5}
\]
The procedure outlined above can be extended to take account of smear and well resistance (Hird et al. 1992).

Unit cell analyses

A series of unit cell analyses was performed, firstly to assess the effect of well resistance and secondly to compare the rate of consolidation of axisymmetric and plane strain unit cells. In these analyses only the soft clay was modelled, Figure 1, and the embankment was represented as vertical loads incrementally applied to correctly model the rate of loading. A total of 100 increments was used, these being divided equally between the loading and consolidation stages.

Axisymmetric unit cell analyses

Two axisymmetric analyses were performed using the mesh shown in Figure 5. In the first analysis drainage elements (Russell 1990) were used to model the Geodrains; the Geodrain discharge capacity was conservatively estimated as 140m³/year (Holtz et al. 1991). The rate of consolidation was compared with that in a second analysis in which the Geodrain was modelled as infinitely permeable; achieved by defining a boundary with zero excess pore pressure.

There was a negligible difference in the rate of consolidation in the two analyses, indicating that well resistance was not significant. The second analysis will be referred to as the axisymmetric analysis in the remainder of the paper.

Plane strain unit cell analyses

As well resistance could be neglected, Equation 5 was used to calculate an equivalent drain spacing for a plane strain unit cell, \( B = 4.5 \text{m} \). The axisymmetric mesh, Figure 5, was proportionally stretched to the required width and used for plane strain analyses.

Two criteria were used to assess the quality of the matching procedure. Firstly, the average value of the excess pore pressure at mid-depth, Figure 6, secondly the average surface settlement,
Figure 6: Unit cell rate of consolidation based on average excess pore pressure at mid-depth.

Figure 7: Unit cell rate of consolidation based on average surface settlement.

Figure 8: Unit cell rate of consolidation matching errors.

Figure 9: Mesh used for full plane strain analysis.

Comparison of finite element and observed behaviour
The large amount of instrumentation at Porto Tolle allowed several comparisons of observed and finite element behaviour. Figure 10 shows the centreline surface settlement with time, Figure 11 the maximum lateral movement beneath the toe with time, Figure 12 the lateral movement profiles at the end of the loading and consolidation stages and Figure 13 the excess pore pressure 19.7m and 12.6m below the surface on the centreline.

All the predicted displacements compare favourably with the observed values. The excess pore water predictions, Figure 13, agree less well with those observed, particularly after construction.

Jamiolkowski and Lancellotta (1984) suggested that during...
Figure 10: Surface settlement at centreline.

Figure 11: Maximum horizontal displacement.

Figure 12a: Lateral movement profile at inclinometer position, end of construction.

Figure 12b: Lateral movement profile at inclinometer position, end of consolidation.

Figure 13a: Excess pore pressure 19.7m below surface on centreline.

Figure 13b: Excess pore pressure 12.6m below surface on centreline.
consolidation the measured pore pressures, although consistent with those from other sections of the trial, did not correctly reflect the progress of consolidation and may have been unreliable. The unsatisfactory long term performance of the piezometers may have been due to the presence of organic gas in the soil.

Conclusion
A methodology for the plane strain analysis of foundations involving vertical drains has been applied to a normally consolidated soft clay. Axisymmetric and plane strain unit cell rates of consolidation, based on excess pore water pressure and surface settlement, are in close agreement, thus validating the procedure. The method has been used in a full plane strain analysis of the Porto Tolle trial embankment for which reliable input parameters and records of observed behaviour were available. Material parameters were assessed from the available literature. Both vertical and lateral displacements predicted by the finite element analysis were in good agreement with those observed. Reasonable agreement between the predicted and observed excess pore pressure was obtained during construction; the poor agreement after construction may be due to errors in the observed readings.

The proposed method of analysis of soils containing vertical drains has been shown to be capable of producing accurate predictions of observed behaviour. The method provides an efficient and versatile procedure for the analysis of soils improved by vertical drains. Careful application of the method will provide a realistic assessment of stability and settlement of embankments constructed on soft soils and allow various other expedients, such as reinforcement and stage construction, to be assessed.

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References

Appendix

Figure 14: Plain strain unit cell.