Mohr Coulomb error correction

by Andrew N Schofield, Cambridge University

Terzaghi and the Mohr Coulomb error

In the 1st ISSMFE Conference (1936) proceedings, Terzaghi writes that newly over-consolidated clay strength fits Mohr's rupture hypothesis (he used the word hypothesis, not criterion). He quotes data for drained shear box tests which his research student Hvorslev fits to the Mohr Coulomb equation. They are wrong. They contradict Coulomb's paper which, in three separate places, states that "l'adhérence est nulle dans les terres nouvellement remuées" (newly worked soil has no cohesion), and they have no data for soil strength on the wet side of critical states (cs). There is no "true" cohesion on the dry side of the critical state. The peak strength of dense soil paste is due to interlocking and friction among the soil particles.

When soil flows, many soil particles change partners, and there is no time to bond particle to particle. It is only when soil is left to age and creep that bonds develop at particle contacts and turn it into soft rock. Renewed strains destroy this strength. Broken bonds do not resist ground failure mechanisms. Bonds are not remade until there has been time for ageing and creep, long after a failure event. Terzaghi's Mohr Coulomb error is to suppose that peak strength seen in Hvorslev's dense, newly remoulded shear test samples indicates strong cohesive bonds, when it really indicates dense packing of particles. **Figure 1** shows how the peak strengths are caused by particle interlocking. Strength in Hvorslev's tests depends on packing geometry, not on chemistry of bonds.

Taylor's interlocking; peak, and ultimate strength of stiff clay

In **Figure 2**, dense soil in a drained box is sheared a distance x by shear force τ. It dilates and lifts the piston (and the normal force σ' that acts on it) by a distance y. Taylor's peak strength of sand in drained shear box tests involves (i) a critical state friction component and (ii) an interlocking component, which increased with increase in the distance of the state of the soil at failure from the ultimate states in the drained test. This applies both to dense sand and stiff clay. In particular, it applies to any paste made by mixing fine particles and water; for example, a mixture of cornflour and water exhibits dilatancy. In Critical State Soil Mechanics (CSSM) 1968, pp232/3, Schofield and Wroth describe North London retaining walls failures, **Figure 3**, and the stiff fissured clay behind them that disintegrated "into a rubble of lubricated blocks, sliding on each other on very thin moist layers of soft, lubricating clay paste". How did those layers become moist and "slick"? That stiff over-consolidated London clay was a paste dilating to a critical state; suction during shearing led it to soften to a critical state after peak strength.

Earth pressure in that rubble of clay blocks satisfies a calculation based on critical state friction because the effective pressure that acts between blocks, and the strength with which they adhere to each other, increases with depth in the rubble; (however, note that care is needed to detect old slip planes where ground in the past had very large displacements; a field shear box test on such a surface may find the strength there to have fallen below the critical state value).

There is no "true" cohesion on the dry side of the critical state. The peak strength of dense soil paste is due to interlocking and friction among the soil particles.

Movement may stop for long periods, and ageing may then begin to create bonds, but renewed deformation of a rubble of fissured clay soon strains a clay paste layer and returns it to the critical state. As intact blocks come under new stress, their strength to resist rupture will always exceed the critical state strength of the paste on their boundaries; CSSM analysed the pressure of stiff fissured London clay on the dry side of critical state in terms of a material with an angle of friction based on the critical state parameter M (denoting capital mu).

"Apparent cohesion" of soil

During critical state flow of soil, the undrained cohesion results from effective stress and critical state friction, not chemical bonds between soil particles. All that small clay mineral particles and chemicals do during steady plastic deformation of soft soil is

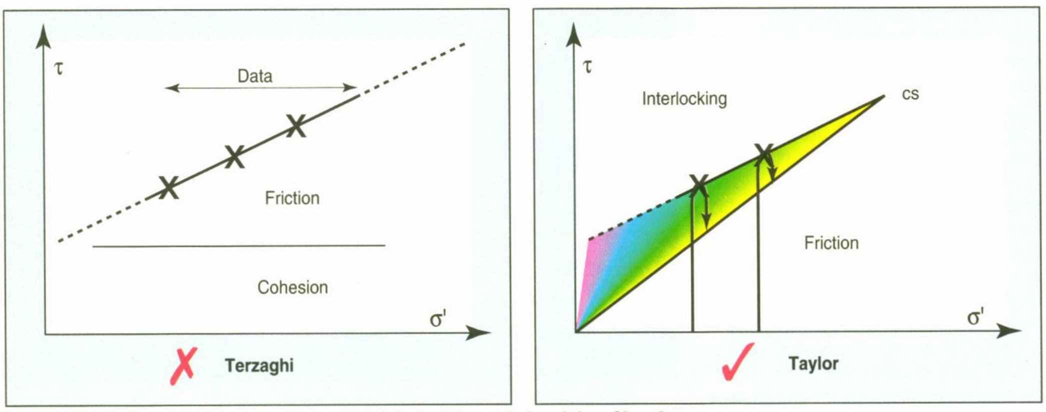


Figure 1: Peak strengths are caused by particle interlocking, not chemistry of bonds.

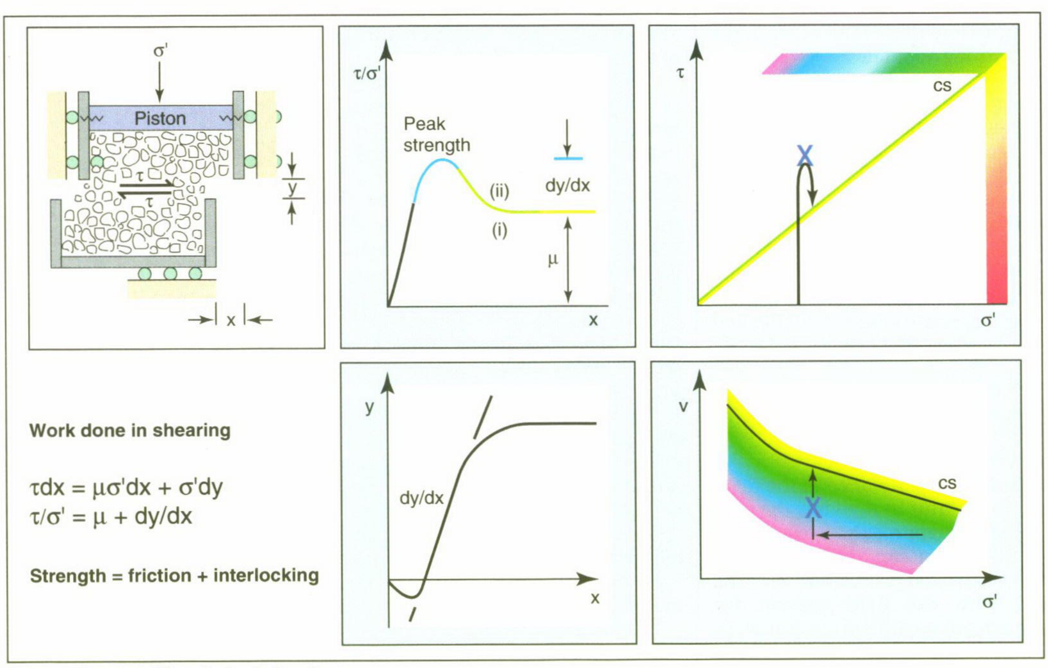


Figure 2: Dense soil in a drained shearbox.
In all figures the spectrum of colours from red to violet indicates strengths at increasingly high stress ratios.

Future correction of the Mohr Coulomb error will invigorate geotechnical teaching, research, and laboratory testing.

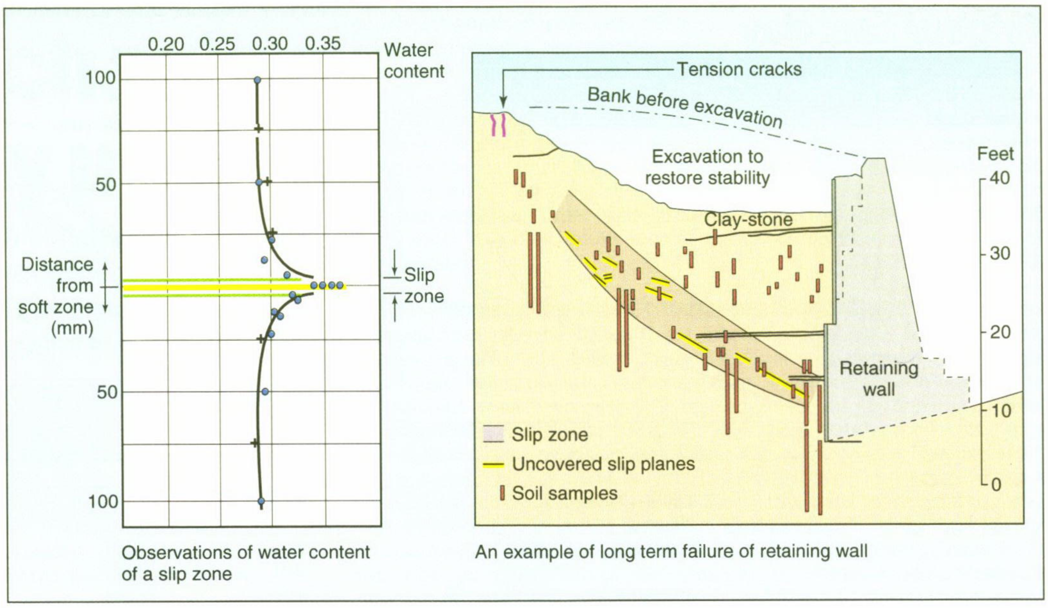


Figure 3: North London retaining walls failure.

to cause a pore water suction, which can be measured. When we remould soft soil in a classification test, its strength is [(suction) x (friction)]. It remains ductile plastic material with constant "apparent cohesion" while it flows at constant volume, because it is at a constant effective stress, and critical state friction is constant. CSSM analyses the bearing capacity of soft clay on the wet side of critical state in terms of a perfectly plastic material with rapid undrained "apparent" cohesion.

The name cam clay asserts that the plastic volume change typical of clay soil behaviour is due to mechanical stability of an aggregate of small, rough, frictional, interlocking hard particles. Coulomb's "newly worked soil" and the cam clay theory give a better insight into true soil behaviour than the erroneous Mohr Coulomb theory and Terzaghi's "true" properties".

In Figure 3 a grey colour indicates soil at stresses below these high peak strengths. A violet colour indicates states in which one normal stress component is zero, and soil may crack.

Cracking and rupture of stiff clay makes ground unstable. Specimens that represent ground in such states inevitably give scattered strength data. Stress circles for specimens at peak strength have an inaccurate envelope on the dry side of critical state; the error in the Mohr Coulomb equation was found by study of yielding on the wet side.

Limiting stress and FE analysis

Two equations of plane equilibrium

$$d\sigma'_x/dx+d\tau/dy=0$$
$$d\tau/dx+d\sigma'_y/dy=0$$

if combined with the Mohr Coulomb equation

$$F(\sigma'_x\sigma'_y\tau)=0$$

give a hyperbolic system of three equations in three unknowns $(\sigma'_x\sigma'_y\tau)$ which have a solution by the method of characteristics for general stress conditions on the boundaries. Terzaghi criticises these Mohr Coulomb solutions for not including strains, but the error lies in his concept of cohesion; the "true" place for strain in the equations is that "cohesion" involves strain.

Many FE calculations exist that do include strain compatibility, but solutions need validation against test data. A site investigation, a laboratory test of "undisturbed" specimens, and a geotechnical analysis in an office using the Drucker Prager or a similar constitutive model, are unconvincing when it is obvious from the increase in water content on any slickensided surface that "cohesive" soil is a dense paste that sucks in water when sheared. Ground engineering makes a basic error when interlocking is neglected in calculations of clay strength.

Critical states

The axial and radial effective stresses σ'_a and σ'_r in a triaxial test are combined to give the mean normal effective pressure

$$p'=(\sigma'_a+2\sigma'_r)$$

and the triaxial deviator stress

$$q=(\sigma'_a-\sigma'_r).)$$

Elastic compression and swelling of test specimens in general follows lines

$$v_\kappa=(v+\kappa\ln p')=\text{const.}$$

which combine pressure p′ and specific volume v to define each aggregation of particles.

Each aggregate, when subject to shear distortion, has a particular critical pressure p'_{crit} at which it will shear at constant volume. Frictional flow of soil in a critical state satisfies an equation

$$q=Mp_{crit}'.$$

In Figure A five different aggregates are represented by five parallel lines. The critical state line (yellow/green) has equation

$$v+\lambda\ln p'_{crit}=\Gamma.$$

Successive stress state points (q,p′,v) in drained and undrained triaxial tests form paths approaching the cs line.

In Figure A the zone to the right of the critical state line where

$$p'>p'_{crit}$$

is called the wet side; shearing there causes aggregates to compress to more dense packing and emit water with ductile stable bulging of a test specimen. To the left of the critical state line, where

$$p'_{crit}>p'$$

is the dry side where shearing causes aggregates to dilate and suck in water and ground slips at peak strength with unstable failures.

At large strain, soil has critical state strength.

For example, a simple cs calculation gives the strength of London clay at 35 per cent water content in Figure 3 from its cs properties; specific gravity 2.75 hence $v=(1+e)=1+(2.75\text{x}0.35)=1.9625$; $\lambda=0.161$, $\kappa=0.062$, $\Gamma=2.759$, $M=0.89$, hence $p'_{crit}=\exp\{(\Gamma-v)/\lambda\}=140.78\text{kN/m}^2$, and $c_u=(0.89\text{x }p'_{crit})/2=62.65\text{kN/m}^2$.

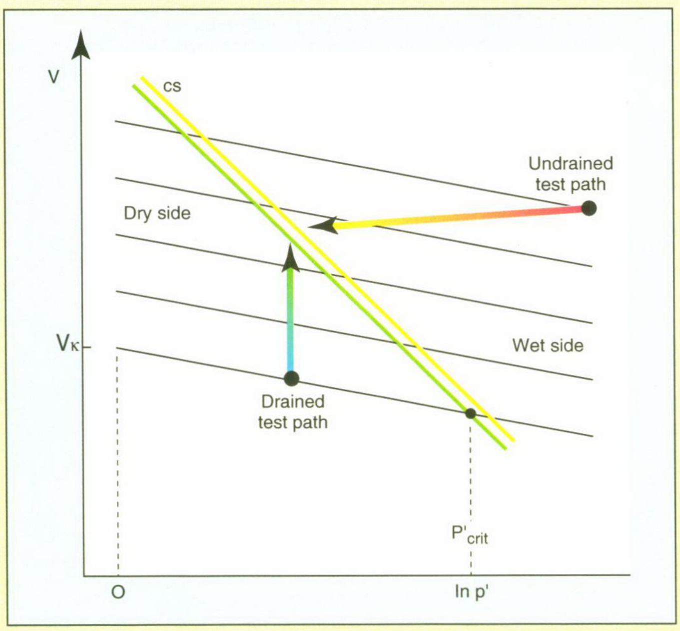


Figure A: Critical states

Original cam clay

An equation is calculated below for a yield locus, Figure B, that defines the resistance of soil to some applied stress (q,p') when $q/p'<M$. This curve has an orange tangent at the point where the stress vector reaches it. When the soil yields and hardens under a stress increment (dq,dp') directed outward from the locus, the yield stress (q,p') is "associated" with the plastic strain rates $(dv_k,d\varepsilon)$ for ductile yielding.

The plastic strain rate vector (associated flow) is normal to the tangent. To ensure stability, outward stress increment vectors must not release energy from ductile stable soil on the wet side of the cs line. The equation

$(dp'dv_\kappa+dqd\varepsilon)\geq 0$, gives

$(dq/dp'+dv_\kappa/d\varepsilon)=0$.

The equation for frictional working by the yield stress during ductile plastic strain is

$p'dv_k+qd\varepsilon=Mp'd\varepsilon$, hence

$(q/p'+dv_\kappa/d\varepsilon)=M$.

Eliminating $dv_k/d\varepsilon$ from these equations and integrating gives the original cam clay equation

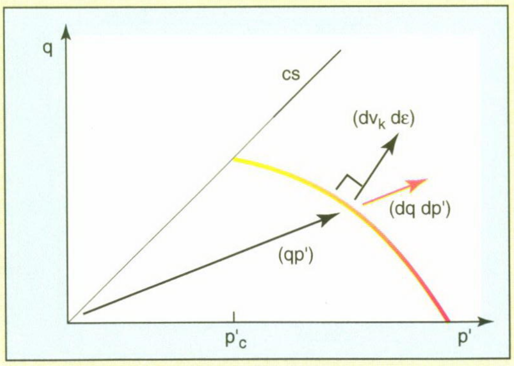
$(q/Mp')=1-\ln(p'/p_c')$.

Confirmation of cam clay theory came from an undrained test of a triaxial specimen on the wet side of cs, reproduced here from CSSM Figures 7.12. Any undrained test has

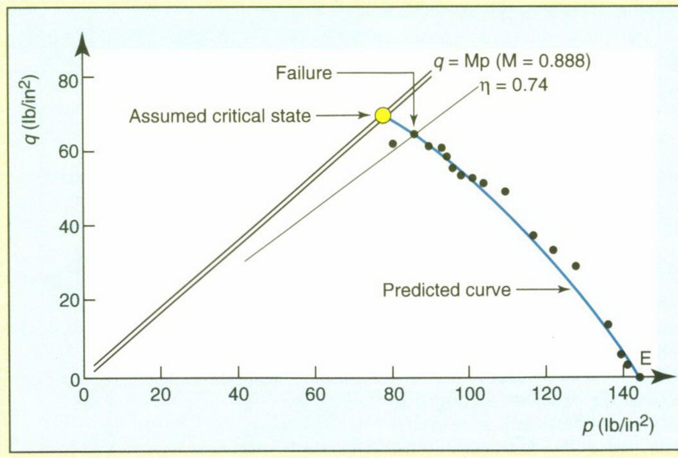
$v=(\text{constant})=v_\kappa-\kappa\ln p'$

which is an inclined line in Figure D. At points along this test path there is an intersection with each yield locus as the specimen yields and hardens.

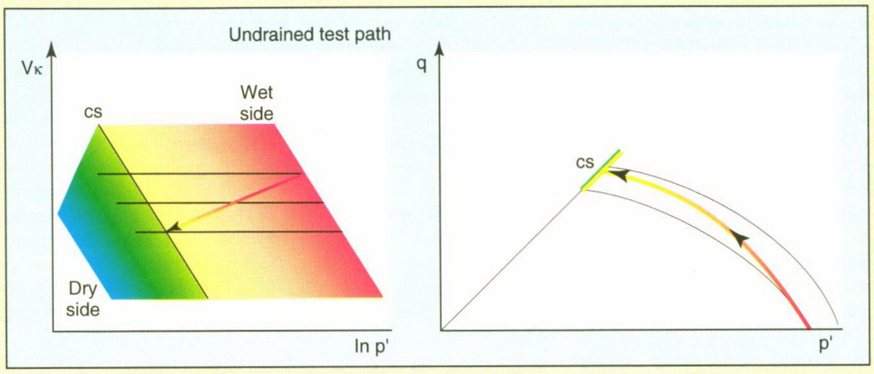
The fact that data points fit the prediction for the undrained test path Figure C (given in detail in CSSM) means there was no cohesion on the wet side of critical state, hence no cohesion on the dry side either.



ABOVE LEFT – Figure B: Yield locus.



ABOVE RIGHT – Figure C: Data points fit the prediction for the undrained test path.



RIGHT – Figure D: Undrained test path.

Correction of the Mohr Coulomb error

Future correction of the Mohr Coulomb error will invigorate geotechnical teaching, research, and laboratory testing.

Soil is an aggregate of small rough frictional interlocking particles with critical state properties that can be determined by classification and triaxial tests. Aggregates more loose (more wet) than the critical state will yield and deform in a ductile stable way described by cam clay theory. If an aggregate is more dense (more dry) than the critical state, it fails in an unstable manner, with rupture planes on which gouge material dilates and softens, or with cracks or channels that suddenly allow rapid transmission of pore water pressure into a soil body (liquefaction). Such facts do not fit the Mohr Coulomb theory; critical state theory fits the facts and solves simple problems. General problems will require observational control of construction, numerical modelling using FE programs based on critical state theory, and physical modelling using centrifuges (now affordable and commercially available, **Figure 4**) to demonstrate failure mechanisms. At present centrifuge models are made of newly remoulded soil paste. New models will be prepared with chemical, thermal, and ageing processes that will create model soft rock bodies.

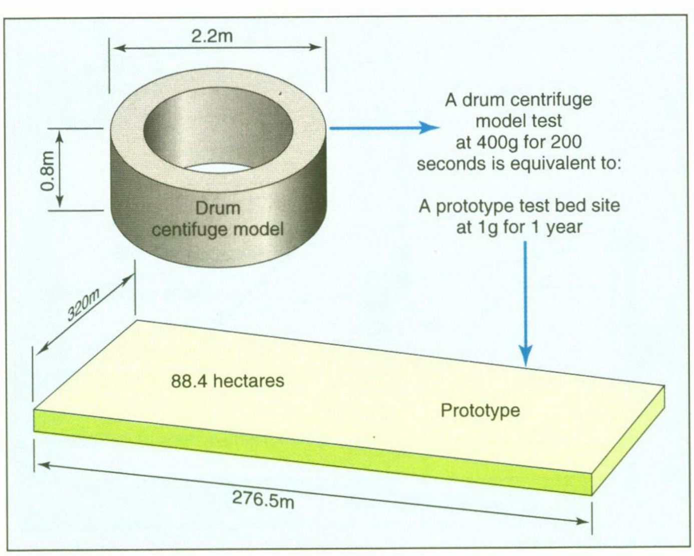


Figure 4: A drum centrifuge.